

$$\frac{\sqrt{1+2x\sqrt{1-x^2}}}{2} + 2x^2 = 1$$

~~$1+2x\sqrt{1-x^2}$~~

Одн  $x \in [-1, 1]$

Пусть  $x$ -решение ур-ия. Тогда найдётся такое  $t$ , что  $x=\sin t$ ,  
тогда  $t \in [-\pi/2; \pi/2]$

$$\begin{aligned} \sqrt{(1+2\sin t\sqrt{1-\sin^2 t})/2} + 2\sin^2 t &= 1 \\ \sqrt{(1+2\sin t\sqrt{\cos^2 t})/2} &= 1 - 2\sin^2 t \\ \sqrt{(1+2\sin t|\cos^2 t|)/2} &= \cos 2t \\ |\cos^2 t| &= \cos^2 t \text{ т.к. } t \in [-\pi/2; \pi/2] \\ \sqrt{(1+2\sin t\cos^2 t)/2} &= \sin(2t+\pi/2) \\ \sqrt{(\sin^2 t + 2\sin t\cos^2 t + \cos^2 t)/2} &= 2\sin(t+\pi/4)\cos(t+\pi/4) \\ |\sin t + \cos t|/\sqrt{2} &= 2\sin(t+\pi/4)\cos(t+\pi/4) \\ |\sin(t+\pi/4)| &= 2\sin(t+\pi/4)\cos(t+\pi/4) \end{aligned}$$

$$\begin{aligned} \sin(t+\pi/4) &= 0 \\ t &= -\pi/4 + Pn \\ \text{т.к. } t &\in [-\pi/2; \pi/2], \text{ то } t = -\pi/4 \end{aligned}$$

1 случай  
 $t \in [-\pi/2; \pi/2]$   
 $-\sin(t+\pi/4) - 2\sin(t+\pi/4)\cos(t+\pi/4) = 0$   
 $\sin(t+\pi/4)(1+2\cos(t+\pi/4)) = 0$   
 $\cos(t+\pi/4) = -\frac{1}{2}$   
 $t = 5\pi/12 + 24Pk/12$   
 $t = -11\pi/12 + 24Pk/12$   
 нет подходящих

2 случай  
 $t \in (-\pi/4; \pi/2]$   
 $\sin(t+\pi/4) - 2\sin(t+\pi/4)\cos(t+\pi/4) = 0$   
 $\sin(t+\pi/4)(1-2\cos(t+\pi/4)) = 0$   
 $\cos(t+\pi/4) = \frac{1}{2}$   
 $t = \pi/12 + 24Pk/12$   
 $t = -7\pi/12 + 24Pk/12$   
 **$t_1 = \pi/12$  при  $k = 0$**

Обратная замена  
 $x = \sin \pi/12 = \sqrt{(1-\cos \pi/6)/2} = \sqrt{(2-\sqrt{3})/2}$   
 $x = \sin(-\pi/4) = -\sqrt{2}/2$

Ответ:  $-\sqrt{2}/2; \sqrt{(2-\sqrt{3})/2}$

$x = \sqrt{\frac{2-\sqrt{3}}{4}}$     $x = -\frac{1}{\sqrt{2}}$